

Hiding Directions, Leaking Structure: Breaking ArrowCloak through Low-Rank Structure

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Abstract—TEE-shielded inference keeps sensitive state in a trusted execution environment (TEE) while offloading linear algebra to an untrusted accelerator. Wang et al., in *Game of Arrows* (USENIX Security 2025), showed that five widely adopted lightweight defenses preserve vector directions and introduced *ArrowMatch* to exploit this leakage. They then proposed *ArrowCloak*, which adds a different multiple of one shared mask direction to each vector and bases its weight-recovery hardness argument on Learning with Errors (LWE). *ArrowCloak* successfully reduces *ArrowMatch* to near-black-box levels.

In this paper, we revisit *ArrowCloak* from cryptographic and structural perspectives. Its LWE formulation does not by itself establish standard LWE hardness: the reduction direction, quantized arithmetic, and joint instance distribution do not meet the required conditions. Reusing one mask direction thus leaves a recoverable rank-one component across the released matrix. We exploit this structure with *ArrowRevelio*, an end-to-end, query-free recovery attack. Given a public checkpoint and the obfuscated weights, *ArrowRevelio* removes the masking subspace, recovers the hidden one-to-one correspondence, and reconstructs protected weights without transformation secrets, victim queries, or fine-tuning data. Across six model-task pairs spanning classification, segmentation, and diffusion, *ArrowRevelio* recovers 99.92–100% of hidden vector correspondences. Reconstructed classification models achieve 94.39–99.54% victim agreement and differ by at most 1.59 percentage points in accuracy; the recovered segmentation model achieves 98.35% output agreement. These findings suggest that lightweight protection should address both per-vector geometry and joint structure across released weights.

I. INTRODUCTION

Deploying proprietary neural-network models on users’ devices enables local, low-latency inference, but it places valuable model weights on hardware controlled by potential adversaries [1], [2]. Unlike a cloud service that exposes only a query interface, on-device deployment gives an adversary a white-box view of the software stack, memory, and hardware interfaces, creating a direct risk of model extraction.

A natural defense is to protect model weights within a trusted execution environment (TEE). However, running an entire modern model inside a CPU TEE forgoes GPU-class acceleration, while GPU TEEs remain unavailable on much of today’s deployed hardware, with mature support largely confined to recent NVIDIA server-grade accelerators [3]. This gap has motivated split-inference systems that keep sensitive

state and lightweight protection logic inside a CPU TEE while outsourcing linear algebra to an untrustworthy accelerator.

Slalom [4] pioneered this design by masking activations and verifying GPU computation. Subsequent systems adapted this split to model confidentiality. Following prior terminology, we call the LLM instance of this architecture *TEE-Shielded LLM Partitioning (TSLP)* [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. These systems either shield selected model components or obfuscate outsourced weights. For weight-obfuscation-based TSLP, keeping trusted-side work cheaper than the matrix multiplication sent to the accelerator favors lightweight per-vector transformations.

Wang et al., in *Game of Arrows* (USENIX Security’25) [17], systematically examined the confidentiality consequences of this low-overhead obfuscation strategy. They showed that these lightweight per-vector schemes, widely adopted in TSLP [7], [8], [12], [13], [14], [15], [16], remain vulnerable to direction-based recovery: their dominant scaling and permutation operations change vector magnitudes and positions but preserve directions. Their analysis assumes models fine-tuned from public checkpoints, with transformed weights exposed to an untrusted GPU while transformation secrets and recovery remain protected by the TEE. Specifically, their *ArrowMatch* attack uses these directions to recover the hidden permutation between public and exposed vectors, then uses query-labeled data to fit vector lengths and construct a surrogate. To counter this attack, the same work introduced *ArrowCloak*, a lightweight direction-hiding defense. *ArrowCloak* scales each vector, adds a vector-specific multiple of one shared mask direction, and permutes the result. The additive component generally changes the exposed direction, disrupting the signal used by *ArrowMatch*, while compact secret state lets the TEE correct the GPU output. *ArrowCloak* further presents an Learning with Errors (LWE)-based security argument for the computational hardness of weight recovery. The authors report that it effectively mitigates *ArrowMatch*, making obfuscated directions nearly as dissimilar as random vectors.

In this paper, we revisit *ArrowCloak* along two complementary dimensions: whether its proposed LWE formulation actually establishes computational hardness, and whether its direction-hiding transformation eliminates exploitable structure in the released weights. We find weaknesses on both fronts, showing that *ArrowCloak* does not provide its intended

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model confidentiality. We first reassess *ArrowCloak*'s proposed connection to LWE. Rewriting an *ArrowCloak* weight relation in an LWE-like form does not transfer LWE hardness: the proposed mapping does not constitute a hardness-preserving reduction from LWE to *ArrowCloak* recovery, its quantized arithmetic fails to preserve the claimed modular relation, and its induced joint distribution over matrices, secrets, and errors does not match standard LWE [19], [20]. These gaps invalidate the stated hardness argument, but do not by themselves imply that *ArrowCloak* can be efficiently broken. We therefore turn to the concrete transformation and ask whether its direction-hiding mechanism nevertheless leaves recoverable structure. We find that it does. Although *ArrowCloak* changes individual vector directions, its masking terms are not independent: they lie in one shared direction in the original construction, or in a shared low-dimensional subspace under the rank- r extension we analyze. Under the conditions we derive, this shared structure can be estimated from the jointly released vectors and projected away, restoring enough of their relationship to the public initialization to recover the hidden permutation.

Based on this insight, we present ***ArrowRevelio***, an efficient, query-free recovery attack against *ArrowCloak*-type protection. Given the public initialization checkpoint, public architecture metadata, and the obfuscated weights exposed to the GPU, *ArrowRevelio* uses singular value decomposition (SVD) to estimate and remove the shared masking subspace, solves a global one-to-one assignment to recover the hidden permutation, and fits the remaining coefficients to reconstruct the victim weights. The attack requires no plaintext victim weights, transformation secrets, victim-model queries, or fine-tuning data. We further derive sufficient spectral-separation conditions for recovering the original rank-one mask direction and its removal projector, and characterize the corresponding failure conditions for rank- r masking. Across six model-task pairs, *ArrowRevelio* achieves 99.92–100% permutation recovery. On ViT-B/16, BERT-base, and GPT-2 classification workloads, the reconstructed models agree with their victims on 94.39–99.54% of test inputs, with task accuracy differing by at most 1.59 percentage points. The recovered segmentation model achieves 98.35% output agreement with its victim, and we additionally evaluate two diffusion workloads.

Viewed together, *Game of Arrows* and our work expose complementary dimensions of leakage in TSLP's weight protection. *ArrowMatch* exploits geometry preserved within individual vectors under scaling and permutation, whereas *ArrowRevelio* exploits structure shared across vectors after *ArrowCloak* conceals this local geometry. This progression shows that preventing per-vector matching is insufficient when the jointly released weights still expose recoverable matrix-level structure. Confidential offloading mechanisms must therefore account for both local geometric leakage and global structure across released weights.

This observation raises a natural design question: can *ArrowCloak* mitigate cross-vector leakage by increasing the mask rank while preserving its lightweight trusted-side design? We study this tradeoff on BERT-base/SST-2 by increasing

the mask rank from 64 to 512 and applying the complete attack with automatic rank estimation. Higher-rank masking progressively weakens functional recovery, but does not eliminate the underlying structural leakage: permutation recovery remains 100% through $r = 256$ and 99.98% at $r = 512$, while victim agreement falls from 95.53% to 48.05%. These results show that higher-rank masking weakens the attack, but requires more trusted-side state and correction work without fully eliminating the underlying structural leakage.

To the best of our knowledge, the only other public critique of *ArrowCloak* is a concurrent work by Chiang et al., released as the *MOSAIC* arXiv preprint on July 31, 2026 [21]. Its Appendix F identifies a flaw in *ArrowCloak*'s LWE-based security argument, but does not evaluate whether the scheme is practically recoverable. Our work audits the argument more broadly, covering its reduction direction, quantized arithmetic, and joint instance distribution. We also present *ArrowRevelio*, the first end-to-end, query-free attack to recover a functional model from *ArrowCloak*-protected weights.

This paper makes the following contributions:

- We audit *ArrowCloak*'s LWE-based security argument and show that it does not transfer LWE hardness, identifying gaps in the reduction direction, quantized arithmetic, and joint instance distribution.
- We uncover shared low-rank masking structure in *ArrowCloak* and develop *ArrowRevelio*, a polynomial-time, query-free recovery attack using only public information and the exposed obfuscated weights. We also derive sufficient conditions for rank-one mask recovery.
- We evaluate *ArrowRevelio* across six model-task pairs, achieving 99.92–100% permutation recovery and recovering victim functionality across classification, segmentation, and diffusion workloads.
- We study higher-rank masking as a mitigation and show that it weakens functional recovery but does not eliminate subspace or permutation recovery, while increasing trusted-side cost.

II. TEE-SHIELDED LLM PARTITIONING

This section introduces the deployment setting and protection mechanism analyzed in this paper. We first describe partitioned inference between a TEE and an untrusted GPU, formalize *ArrowCloak*'s weight transformation and efficient output recovery, and summarize its claimed security argument. Table I summarizes the notation conventions used throughout the paper; problem-specific symbols are defined when they are first introduced.

A. Partitioned Inference with an Untrusted GPU

A trusted execution environment (TEE) isolates selected code and data from the rest of the system, including privileged host software [22]. This isolation can protect proprietary model state on a user-controlled device. Executing an entire model inside a CPU TEE is costly because CPU TEEs lack GPU-class GEMM throughput and incur additional trusted-memory and data-movement overhead. Prior work therefore partitions

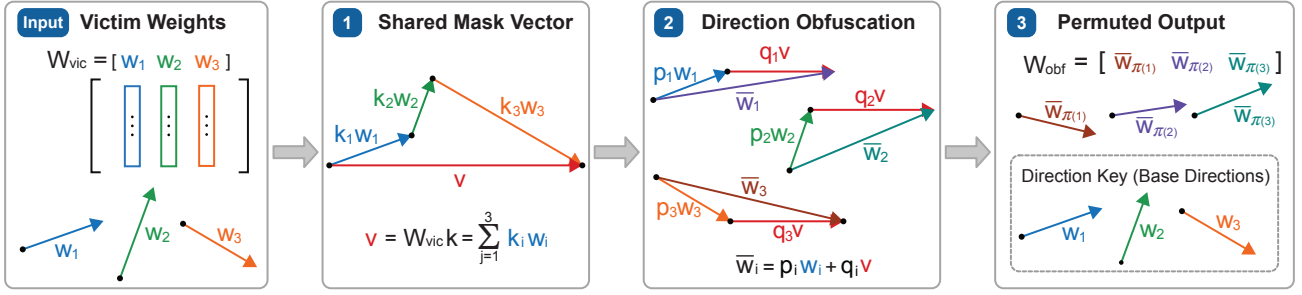


Fig. 1. *ArrowCloak* counters *ArrowMatch* by adding a shared mask vector to rotate victim-weight vectors before permutation.

TABLE I
NOTATION USED THROUGHOUT THE PAPER.

Symbol	Meaning
A, W, H, Π	Matrices
$\mathbf{x}, \mathbf{v}, \mathbf{q}$	Column vectors
p, q, i	Scalars or indices
$A_{:,i}, A_{i,:}, A^\top, A^{-1}$	Column i , row i , transpose, and inverse of A
$I, \mathbf{1}, \mathbf{e}_i, \text{diag}(\cdot), \lfloor \cdot \rfloor$	Identity, ones, basis vector, diagonalization, and nearest-integer rounding
$\ \cdot\ _2, \ \cdot\ _F, \cos(\mathbf{x}, \mathbf{y})$	Euclidean/spectral norm, Frobenius norm, and cosine similarity

machine-learning workloads between a CPU TEE and a faster, untrusted accelerator, typically retaining sensitive state and lightweight operations inside the TEE while outsourcing expensive linear algebra [4], [23], [24].

TEE-Shielded LLM Partitioning (TSLP) [7], [8], [12], [13], [14], [15], [16] applies this design to on-device model inference. For an offloaded linear layer, the TEE transforms the private weights and exposes only the transformed weights to the GPU. The GPU performs the dominant matrix multiplication, after which the TEE recovers the output of the original layer. Other operations that are not suited to outsourcing remain inside the TEE. Unlike systems that extend trusted execution directly to the accelerator [25], [26], this setting treats the GPU and its memory as observable by the device owner.

The weight transformation must satisfy competing security and efficiency requirements. It should prevent an observer of the GPU from recovering useful private weights, while its preparation and output-recovery costs must remain substantially below the outsourced matrix multiplication. Consequently, lightweight TSLP schemes commonly use vector-wise scaling, permutation, and structured masking rather than dense secret transformations.

B. From *ArrowMatch* to *ArrowCloak*

Among recent proposals for lightweight weight protection in TSLP, *Game of Arrows* introduces *ArrowCloak* [17], a scheme intended to conceal weight-vector directions while retaining inexpensive TEE-side recovery. The scheme is developed in response to *ArrowMatch*, an attack that exposes direction leakage in earlier lightweight obfuscation mechanisms.

Specifically, let W_{pre} denote the weights of a public pre-trained model and W_{vic} the corresponding weights after private fine-tuning. The attacker knows W_{pre} and observes the transformed weight vectors derived from W_{vic} and exposed to the untrusted GPU. *ArrowMatch* observes that corresponding weight vectors in W_{pre} and W_{vic} often remain directionally similar. Because scaling preserves vector direction and permutation only reorders the vectors, neither operation removes this similarity. An attacker can therefore compare exposed vectors with those in W_{pre} and use their cosine similarities to infer the hidden matching. It can then reverse the remaining lightweight transformations and construct a surrogate model [17].

ArrowCloak targets precisely the invariant exploited by *ArrowMatch*: before permuting the weight vectors, it rotates each one away from its original direction by adding a mask. Figure 1 illustrates this input and a three-step transformation, which we formalize below:

Input: Victim Weights. Write $W_{\text{vic}} = [\mathbf{w}_1 \cdots \mathbf{w}_n] \in \mathbb{R}^{d \times n}$, where each arrow in the input panel depicts the direction of one victim-weight column $\mathbf{w}_i$.

Step ①: Shared Mask Vector. *ArrowCloak* samples $\mathbf{k} = (k_1, \dots, k_n)^\top$ and forms the shared mask vector

$$\mathbf{v} = W_{\text{vic}} \mathbf{k} = \sum_{j=1}^n k_j \mathbf{w}_j. \quad (1)$$

The red vector in step ① is this shared mask vector, later added to each column with a column-specific coefficient. Following *ArrowCloak*'s scheme description, each coefficient k_i takes values in $\mathbb{Z}_u$, where u is a large prime.

Step ②: Direction Obfuscation. For each column, *ArrowCloak* samples a positive scale p_i and a mask coefficient q_i , then computes

$$\bar{\mathbf{w}}_i = p_i \mathbf{w}_i + q_i \mathbf{v}. \quad (2)$$

The scaling term $p_i \mathbf{w}_i$ remains collinear with $\mathbf{w}_i$; adding $q_i \mathbf{v}$ changes the direction of the exposed vector, thereby removing the directional invariant exploited by *ArrowMatch*.

Step ③: Permuted Output. Define $D_1 = \text{diag}(p_1, \dots, p_n)$ and $\mathbf{q} = (q_1, \dots, q_n)^\top$. Stacking the transformed columns gives $\bar{W} = W_{\text{vic}} D_1 + \mathbf{v} \mathbf{q}^\top = W_{\text{vic}} (D_1 + \mathbf{k} \mathbf{q}^\top)$. *ArrowCloak* then samples a permutation matrix Π and exposes

$$W_{\text{obf}} = \bar{W} \Pi = W_{\text{vic}} H \Pi, \quad H = D_1 + \mathbf{k} \mathbf{q}^\top. \quad (3)$$

Here, Π only reorders the rotated columns.

Efficient recovery. For an input matrix X , the GPU returns $Y_{\text{obf}} = XW_{\text{obf}}$. The TEE knows D_1 , q , Π , and v and recovers the private layer output as

$$Y_{\text{vic}} = Y_{\text{obf}}\Pi^{-1}D_1^{-1} - (Xv)q^\top D_1^{-1}. \quad (4)$$

The only multiplication involving the private input and mask is the matrix–vector product Xv ; the remaining operations are scaling, replication, and permutation. Accordingly, *ArrowCloak* characterizes its online TEE cost as linear in the relevant vector dimensions, avoiding a dense matrix–matrix multiplication. For our later analysis, we also consider a natural rank- r extension that replaces the rank-one term $kq^\top$ with $KR^\top$, thereby introducing r shared mask directions.

Claimed security argument. Beyond disrupting the directional similarity exploited by *ArrowMatch*, *ArrowCloak* further motivates its security through a claimed connection to matrix Learning with Errors (LWE). It argues that, after quantization, recovering the hidden transformation from the public and obfuscated weights constitutes an LWE problem and is therefore computationally hard. In Section III, we further formalize and audit this claim.

III. REASSESSING THE LWE-BASED SECURITY CLAIM

In this section, we formalize and reassess *ArrowCloak*’s LWE-based security. We first review standard LWE and examine the reduction required to transfer its hardness to *ArrowCloak* recovery. We then reconstruct *ArrowCloak*’s claimed mapping and examine the problems in its reduction direction, arithmetic, instance distributions, and parameter requirements.

A. LWE and the Required Security Argument

Fix a dimension n , a modulus $q \geq 2$, and an error distribution χ over $\mathbb{Z}$. An LWE sample for a secret $s \in \mathbb{Z}_q^n$ is obtained by drawing $a \leftarrow \mathcal{U}(\mathbb{Z}_q^n)$ and $e \leftarrow \chi$ independently and releasing

$$(a, b), \quad b = \langle a, s \rangle + e \bmod q. \quad (5)$$

Stacking m independent samples gives $A \leftarrow \mathcal{U}(\mathbb{Z}_q^{m \times n})$ and $b = As + e \bmod q$ with $e \leftarrow \chi^m$. The search-LWE problem asks to recover s from (A, b) [19], [20].

LWE is believed to be computationally hard for several choices of secret and error distributions and underlies many lattice-based cryptographic constructions [19], [20]. By contrast, Bootle et al. [27] show that the integer variant obtained by omitting modular reduction (ILWE) is efficiently solvable under mild conditions on the coefficient and error distributions and their relative variances, using least-squares regression.

To base the hardness of a recovery problem $\mathcal{P}$ on LWE, one must show that an efficient solver for $\mathcal{P}$ would imply an efficient solver for LWE. For a direct recovery reduction, this can be done by mapping an LWE challenge to a valid instance of $\mathcal{P}$ such that solving the latter enables solving the original LWE challenge. Section III-B examines whether *ArrowCloak* provides such a reduction.

B. Auditing ArrowCloak’s LWE Mapping

ArrowCloak starts from the real-valued identity

$$W_{\text{pre}} = W_{\text{obf}}\tilde{H}^{-1} + (W_{\text{pre}} - W_{\text{vic}}), \quad (6)$$

and associates W_{pre} with the LWE observation, W_{obf} with the public coefficient matrix, $\tilde{H}^{-1}$ with the secret, and $E = W_{\text{pre}} - W_{\text{vic}}$ with the error. It then rounds these real-valued matrices separately into a finite field and argues that Eq. (6) reduces *ArrowCloak* recovery to LWE.

To justify treating $E = W_{\text{pre}} - W_{\text{vic}}$ as the LWE error term, *ArrowCloak* examines its empirical distribution using weight histograms, Kolmogorov–Smirnov tests, and its measured standard deviation [17]. These checks concern only whether the fine-tuning residual resembles the proposed error distribution. Together, the algebraic correspondence, quantization step, and empirical residual analysis form the basis of *ArrowCloak*’s LWE-based security argument.

We next identify the gaps that prevent this argument from establishing the claimed hardness.

Reduction in the wrong direction. As discussed in Section III-A, transferring LWE hardness requires a reduction from LWE to *ArrowCloak* recovery. Instead, *ArrowCloak* starts from its own weight-transformation relation and rewrites it in LWE-like notation. It provides no reduction showing how an efficient adversary that breaks *ArrowCloak* can be transformed into an algorithm that breaks the corresponding LWE hardness assumption, and therefore does not establish *ArrowCloak* recovery hardness from LWE hardness.

The absence of this reduction is already sufficient to invalidate the claimed hardness transfer. Even with a reduction in the required direction, its output would still need to be a valid LWE instance. The proposed mapping fails both arithmetically and distributionally.

Incorrect application of quantization. In *ArrowCloak*’s proposed mapping, the integer matrices in the claimed LWE instance are obtained from Eq. (6) using the common scale $\Delta = 2^m$:

$$\begin{aligned} A^\top &= \lfloor \Delta W_{\text{obf}} \rfloor, & S &= \lfloor \Delta \tilde{H}^{-1} \rfloor, \\ B &= \lfloor \Delta W_{\text{pre}} \rfloor, & E &= \lfloor \Delta (W_{\text{pre}} - W_{\text{vic}}) \rfloor. \end{aligned}$$

Thus, even before accounting for rounding error,

$$A^\top S \approx \Delta^2 W_{\text{obf}} \tilde{H}^{-1}, \quad B - E \approx \Delta W_{\text{obf}} \tilde{H}^{-1}.$$

Hence, $B = A^\top S + E \pmod{q}$ does not follow from the original real-valued identity: the product contains an additional factor of Δ . Reduction modulo q does not remove this mismatch. A valid fixed-point encoding would need to specify a rescaling operation, such as multiplication by Δ^{-1} in $\mathbb{Z}_q$, and account for the rounding errors induced by both quantization and multiplication. The published mapping specifies neither step.

Mismatch with LWE distributions. In *ArrowCloak*, the quantized matrices A and E are derived from $W_{\text{vic}}H\Pi$ and $W_{\text{pre}} - W_{\text{vic}}$, respectively, and therefore share dependence on the victim weights. The quantized secret S is derived

from $\Pi^{-1}H^{-1}$, whose structure is determined by H and Π . However, in standard matrix LWE, A should be uniform, E should be sampled independently from a specified small distribution, and S should follow the secret distribution of the invoked LWE variant [19], [20]. Consequently, the published mapping does not establish that its induced instance follows an LWE distribution. Moreover, *ArrowCloak*'s empirical checks concern only the marginal distribution of E and therefore do not establish that the joint distribution of (A, S, E) matches the invoked LWE variant.

Taken together, these gaps prevent *ArrowCloak*'s security argument from establishing recovery hardness based on LWE.

IV. *ArrowRevelio*: A STRUCTURAL RECOVERY ATTACK

Having shown in Section III that *ArrowCloak*'s LWE mapping does not establish recovery hardness, we now ask whether its concrete construction can nevertheless be broken efficiently. Inspecting the transformation reveals that its lightweight masking mechanism reuses a shared direction across protected columns, leaving a rank-one structure in the exposed weights. We exploit this structure with *ArrowRevelio*, a query-free recovery attack that uses only the public pre-trained weights W_{pre} and the exposed obfuscated weights W_{obf} . The attack estimates and removes the shared mask direction, recovers the hidden permutation, and reconstructs the protected weights. We present the rank-one attack first, then extend it to rank- r masking and analyze its computational complexity.

A. Threat Model and Attack Overview

Threat model. Following the identical threat model in *ArrowCloak* [17], we assume an on-device deployment scenario and an honest-but-curious adversary model. The model user follows the prescribed inference protocol but controls the host and untrusted GPU, and thus observes the model architecture, tensor metadata, and obfuscated weights W_{obf} . Our main setting assumes access to the exact public checkpoint W_{pre} from which the private victim weights W_{vic} were fine-tuned. The adversary cannot access W_{vic} , transformation secrets, private training state, or TEE-protected data, and does not exploit TEE vulnerabilities, side channels, or protocol deviations. The adversary aims to construct a surrogate model $\hat{M}$ that matches M_{vic} in behavior and task performance. Exact recovery of every victim weight or transformation secret is unnecessary.

Inspiration: Shared-direction leakage. A low-rank mask is introduced in *ArrowCloak* to perturb the victim model weight. Recall from Section II that it transforms a victim weight matrix as

$$W_{\text{obf}} = (W_{\text{vic}}D_1 + \mathbf{v}\mathbf{q}^\top)\Pi, \quad (7)$$

The additive term $\mathbf{q}_i\mathbf{v}$ changes the direction of column i before permutation, but all such terms share the same mask direction $\mathbf{v}$. Thus, for nonzero $\mathbf{v}$ and $\mathbf{q}$,

$$\text{rank}(\mathbf{v}\mathbf{q}^\top\Pi) = 1, \quad \text{col}(\mathbf{v}\mathbf{q}^\top\Pi) = \text{span}(\mathbf{v}). \quad (8)$$

The hidden permutation Π reorders the columns but does not conceal the shared direction of the additive masks.

The transformation introduces an observable low-rank structure in W_{obf} , which may be used to remove the effect of the mask direction $\mathbf{v}$. Specifically, $\mathbf{v}$ could be estimated by SVD, in which the leading left singular direction of centered W_{obf} estimates $\text{span}(\mathbf{v})$.

Projecting out this direction suppresses the shared mask. Because $W_{\text{vic}} = W_{\text{pre}} - E$, projected public and obfuscated columns linked by the hidden permutation remain approximately collinear when the projected fine-tuning residual is small. Therefore, we can use one-to-one column matching to find the hidden permutation and recover the victim model weight.

Attack overview. Figure 2 summarizes how *ArrowRevelio* turns this observation into a four-stage recovery attack. First, it estimates the shared mask direction from W_{obf} . Second, it projects this direction out of both the public and obfuscated weights. Third, it matches the projected columns through a global one-to-one assignment to recover the hidden permutation. Finally, using this permutation, it fits each column's scale and effective mask coefficient to reconstruct the victim weights.

B. Estimating the Shared Direction

We now show how to estimate the shared mask direction from W_{obf} . Although the rank-one mask component is mixed with the scaled victim weights, its direction can be estimated from the leading left singular vector of the centered exposed matrix. We first center the exposed columns by subtracting their mean. Define

$$\boldsymbol{\mu}_{\text{obf}} = \frac{1}{n}W_{\text{obf}}\mathbf{1}, \quad \widetilde{W}_{\text{obf}} = W_{\text{obf}} - \boldsymbol{\mu}_{\text{obf}}\mathbf{1}^\top. \quad (9)$$

Thus, the columns of $\widetilde{W}_{\text{obf}}$ have zero mean. This centering makes the subsequent SVD depend on variation across columns rather than on their overall mean.

Let

$$\widetilde{W}_{\text{obf}} = U\Sigma V^\top \quad (10)$$

be the SVD of the centered exposed matrix, with singular values in non-increasing order. For the rank-one construction, the attacker estimates the mask direction using the leading left singular vector,

$$\hat{\mathbf{u}} = U_{:,1}. \quad (11)$$

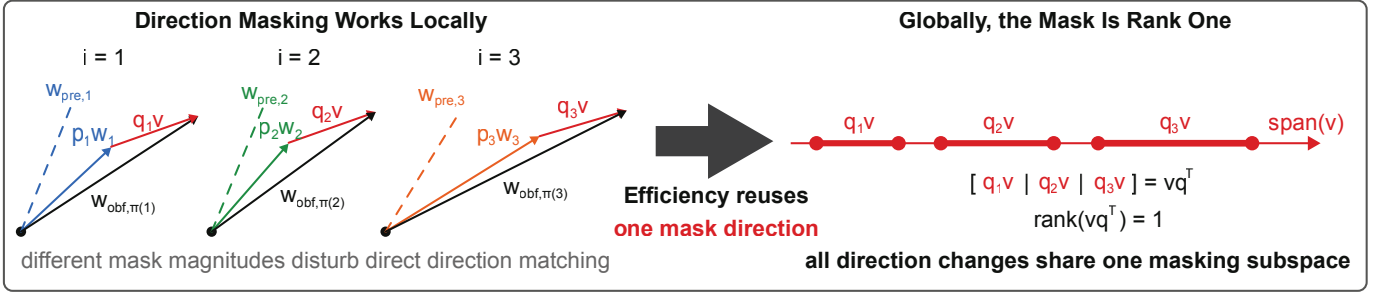
The sign of $\hat{\mathbf{u}}$ is immaterial because the attack uses only its span.

Why it works. For compactness in the analysis, we write the same centering operation as

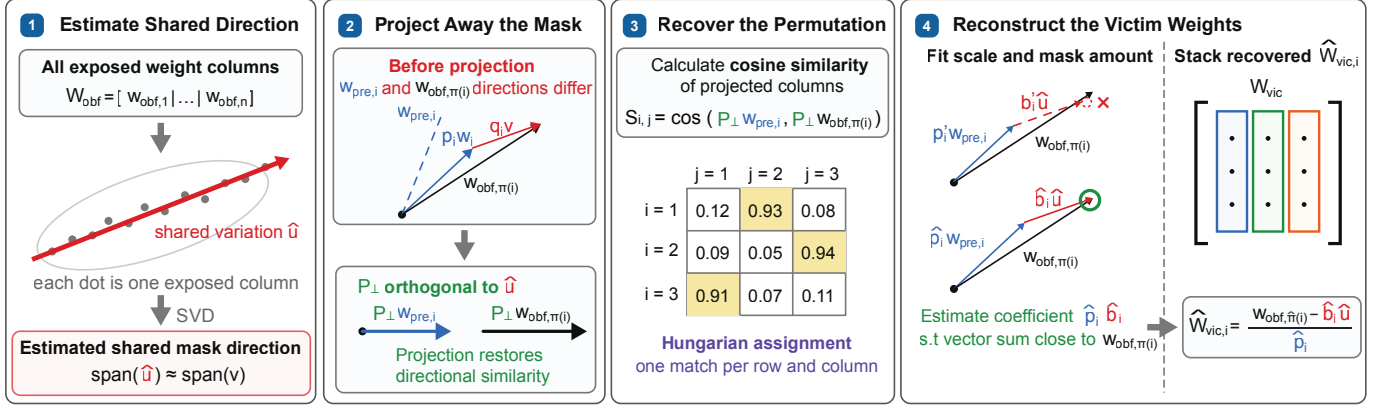
$$\widetilde{W}_{\text{obf}} = W_{\text{obf}}P_{1^\perp}, \quad P_{1^\perp} = I - \frac{1}{n}\mathbf{1}\mathbf{1}^\top.$$

To explain the estimator, define the following latent quantities for analysis:

$$\begin{aligned} X &= W_{\text{vic}}D_1, & \widetilde{X} &= XP_{1^\perp}, \\ \tilde{\mathbf{q}} &= P_{1^\perp}\mathbf{q}, & M &= \mathbf{v}\tilde{\mathbf{q}}^\top. \end{aligned} \quad (12)$$



(a) Shared-direction leakage. The additive masks applied to individual columns all lie in one shared subspace.



(b) ArrowRevelio attack workflow. The attack estimates and removes the shared mask subspace, recovers the hidden column permutation through one-to-one assignment, and fits the remaining coefficients to reconstruct the victim weights.

Fig. 2. Shared-direction leakage and the ArrowRevelio attack workflow.

Because a permutation preserves the all-ones vector, $\Pi P_{1\perp} = P_{1\perp} \Pi$. Applying the centering operator to Eq. (7) therefore gives

$$\begin{aligned} \tilde{W}_{\text{obf}} &= (X + v q^T) \Pi P_{1\perp} \\ &= (X P_{1\perp} + v \tilde{q}^T) \Pi \\ &= (\tilde{X} + M) \Pi. \end{aligned} \quad (13)$$

We first analyze the rank-one mask signal M in isolation. Each column of M is a scalar multiple of v , so M has rank at most one. If $v \neq 0$ and $\tilde{q} \neq 0$, at least one column is nonzero and M has rank one. To identify its leading direction, for $u = v/\|v\|_2$, let $\lambda_{\text{mask}} = \|\tilde{q}\|_2^2 \|v\|_2^2 > 0$. Direct multiplication gives

$$M M^T u = \|\tilde{q}\|_2^2 v v^T \frac{v}{\|v\|_2} = \lambda_{\text{mask}} u. \quad (14)$$

Thus, u is an eigenvector with positive eigenvalue λ_{mask} . Because M is nonzero and rank one, $M M^T$ is a nonzero rank-one positive-semidefinite matrix. Therefore, u spans its unique leading eigenspace, and the leading unit eigenvector is u up to sign.

We now return to the observed centered matrix $\tilde{W}_{\text{obf}}$. Since Π is a permutation matrix, $\Pi \Pi^T = I$. Therefore,

$$\begin{aligned} \tilde{W}_{\text{obf}} \tilde{W}_{\text{obf}}^T &= (\tilde{X} + M) \Pi \Pi^T (\tilde{X} + M)^T \\ &= (\tilde{X} + M)(\tilde{X} + M)^T. \end{aligned} \quad (15)$$

The leading left singular vector of $\tilde{W}_{\text{obf}}$ is the leading eigenvector of this Gram matrix.

Relative to the isolated mask signal, the scaled victim component $\tilde{X}$ and the resulting cross terms act as a spectral perturbation. When that perturbation is small relative to λ_{mask} , the leading left singular vector $\hat{u}$ approaches u up to sign. Section V formalizes this condition and the resulting direction-recovery guarantee.

C. Recovering the Hidden Permutation

We estimate the hidden permutation in two steps. We first show how to derive the projected column relation under the ideal mask-removal projector. We then use the estimated projector to construct a one-to-one assignment.

Ideal mask removal. We remove v mask by projections. Let

$$u = \frac{v}{\|v\|_2}, \quad P_{v^\perp} = I - u u^T. \quad (16)$$

The matrix $P_{v^\perp}$ is the orthogonal projector onto the hyperplane $v^\perp$ and satisfies $P_{v^\perp} v = 0$. Recall that

$$W_{\text{obf}} = (W_{\text{vic}} D_1 + v q^T) \Pi. \quad (17)$$

Projecting the exposed matrix onto $v^\perp$ removes the rank-one mask:

$$\begin{aligned} P_{v^\perp} W_{\text{obf}} &= P_{v^\perp} (W_{\text{vic}} D_1 + v q^T) \Pi \\ &= (P_{v^\perp} W_{\text{vic}} D_1) \Pi. \end{aligned} \quad (18)$$

Because $W_{\text{vic}} = W_{\text{pre}} - E$, its i th column satisfies $W_{\text{vic},i} = W_{\text{pre},i} - E_{:,i}$. Recall that $D_1 = \text{diag}(p_1, \dots, p_n)$ with positive scales $p_i > 0$. Let $\pi(i)$ denote the exposed position assigned to victim column i . Since projection removes the shared mask component, the corresponding projected column satisfies

$$P_{v^\perp} W_{\text{obf},\pi(i)} = p_i (P_{v^\perp} W_{\text{pre},i} - P_{v^\perp} E_{:,i}). \quad (19)$$

Because $p_i > 0$, when $\|P_{v^\perp} E_{:,i}\|_2 \ll \|P_{v^\perp} W_{\text{pre},i}\|_2$, the correct projected pair is nearly positively aligned and its oracle cosine similarity is close to one. This motivates matching the projected columns by cosine similarity.

Estimated projection and global assignment. In practice, the true direction $\mathbf{u}$ is unavailable. Using the estimate $\hat{\mathbf{u}}$ from Eq. (11), the attacker constructs

$$P_{\hat{\mathbf{u}}^\perp} = I - \hat{\mathbf{u}}\hat{\mathbf{u}}^\top. \quad (20)$$

Let

$$S_{ij} = \cos(P_{\hat{\mathbf{u}}^\perp} W_{\text{pre},i}, P_{\hat{\mathbf{u}}^\perp} W_{\text{obf},j}), \quad (21)$$

be the projected cosine similarity between public column i and exposed column j . Since π is bijective, every public column must be assigned to exactly one exposed column, and every exposed column must be used exactly once. We therefore solve

$$\hat{\pi} = \arg \max_{\sigma \in S_n} \sum_{i=1}^n S_{i,\sigma(i)}. \quad (22)$$

We solve this one-to-one linear-assignment problem using the Hungarian algorithm [28] applied to $-S$. The Hungarian algorithm enforces the one-to-one constraint globally, yielding a valid permutation.

Using $P_{\hat{\mathbf{u}}^\perp}$ instead of $P_{v^\perp}$ introduces an additional score perturbation beyond the fine-tuning residual in Eq. (19). Section V-C bounds the projector error, and Appendix A shows that the assignment exactly recovers π when the projected columns retain nonzero norm and the correct cosine matches have sufficient margin.

D. Reconstructing the Victim Weights

Given the recovered permutation and mask direction, we estimate each column's scale and effective mask coefficient by constrained least squares and then invert the fitted transformation.

Coefficient estimation. For a correctly matched column, such that $\hat{\pi}(i) = \pi(i)$, the exposed column satisfies

$$\begin{aligned} W_{\text{obf},\hat{\pi}(i)} &= p_i W_{\text{vic},i} + q_i \mathbf{v} \\ &= p_i W_{\text{pre},i} + q_i \mathbf{v} - p_i E_{:,i}. \end{aligned} \quad (23)$$

Because $\hat{\mathbf{u}} \approx \pm \mathbf{v} / \|\mathbf{v}\|_2$ and the fine-tuning residual is small, we use the approximation

$$W_{\text{obf},\hat{\pi}(i)} \approx p_i W_{\text{pre},i} + b_i \hat{\mathbf{u}}, \quad (24)$$

where b_i absorbs the mask coefficient, the norm of $\mathbf{v}$, and the sign ambiguity of $\hat{\mathbf{u}}$. We estimate the two coefficients by constrained least squares:

$$(\hat{p}_i, \hat{b}_i) = \arg \min_{p \in [p_{\min}, p_{\max}], b \in \mathbb{R}} \|W_{\text{obf},\hat{\pi}(i)} - p W_{\text{pre},i} - b \hat{\mathbf{u}}\|_2^2. \quad (25)$$

The interval $[p_{\min}, p_{\max}]$ enforces the scaling range used by *ArrowCloak*, while b is unconstrained because it absorbs the unknown mask magnitude and sign.

Weight reconstruction. Using the fitted coefficients, we reconstruct the victim column as

$$\widehat{W}_{\text{vic},i} = \frac{W_{\text{obf},\hat{\pi}(i)} - \hat{b}_i \hat{\mathbf{u}}}{\hat{p}_i}. \quad (26)$$

Although the public column is used to estimate the two coefficients, the reconstruction uses the original exposed column. The fine-tuning residual in Eq. (23) is therefore retained after mask removal rather than replaced by zero. The remaining reconstruction error reflects permutation-recovery errors, scale-estimation error, and residual mask error after subtracting $\hat{b}_i \hat{\mathbf{u}}$.

E. Rank- r Extension

The preceding subsections focus on the rank-one case, but our attack extends naturally to a rank- r mask by replacing the single shared direction with an r -dimensional mask subspace.

Rank- r mask structure. Following the rank- r construction introduced in Section II, the additive mask $(W_{\text{vic}} K) R^\top$ has rank at most r and admits a factorization $BC^\top$, where $B \in \mathbb{R}^{d \times r}$ has orthonormal columns and $C \in \mathbb{R}^{n \times r}$ contains the corresponding coefficients. Thus,

$$\begin{aligned} \bar{W} &= W_{\text{vic}} D_1 + (W_{\text{vic}} K) R^\top \\ &= W_{\text{vic}} D_1 + BC^\top, \end{aligned} \quad (27)$$

$$W_{\text{obf}} = \bar{W} \Pi.$$

Thus, the columns of B form an orthonormal basis for the shared mask subspace, and the rows of C contain the column-specific mask coefficients. As in the rank-one case, the permutation only reorders the exposed columns and does not alter the shared mask subspace.

Let $\tilde{X} = W_{\text{vic}} D_1 P_{1^\perp}$ and $\tilde{C} = P_{1^\perp} C$. Applying the same column centering as in Eq. (9) gives

$$\tilde{W}_{\text{obf}} = (\tilde{X} + B \tilde{C}^\top) \Pi. \quad (28)$$

Automatic rank inference. When the mask rank is not available from the scheme configuration, the attacker estimates it from the exposed spectrum. Let s_j denote the j th singular value in the SVD of $\tilde{W}_{\text{obf}}$. The estimator examines the adjacent singular-value ratios

$$g_j = \frac{s_j}{s_{j+1}}, \quad \hat{r} = \arg \max_j g_j. \quad (29)$$

Let $g_{(1)}$ and $g_{(2)}$ denote the largest and second-largest ratios, respectively. The estimator accepts $\hat{r}$ only if

$$g_{(1)} \geq \tau_{\text{gap}} \quad \text{and} \quad \frac{g_{(1)}}{g_{(2)}} \geq \tau_{\text{sep}}, \quad (30)$$

where τ_{gap} controls the minimum break magnitude and τ_{sep} controls its separation from competing breaks.

Mask-subspace estimation and removal. Instead of using the single estimated direction $\hat{\mathbf{u}}$, the attacker uses the SVD in Eq. (10) to define

$$\hat{B} = [U_{:,1} \mid \dots \mid U_{:,\hat{r}}] \in \mathbb{R}^{d \times \hat{r}}. \quad (31)$$

The columns of $\widehat{B}$ are orthonormal and span an estimate of $\text{col}(B)$. It then replaces the rank-one projector with

$$P_{\text{span}(\widehat{B})^\perp} = I - \widehat{B}\widehat{B}^\top. \quad (32)$$

Matching and reconstruction. Matching proceeds on the projected columns as in Section IV-C. After matching, the attack replaces the scalar term $b\widehat{u}$ with $\widehat{B}\widehat{c}$ and fits

$$(\widehat{p}_i, \widehat{c}_i) = \arg \min_{\substack{p \in [p_{\min}, p_{\max}], \\ \mathbf{c} \in \mathbb{R}^{\widehat{r}}}} \|W_{\text{obf}, \widehat{\pi}(i)} - pW_{\text{pre}, i} - \widehat{B}\widehat{c}\|_2^2. \quad (33)$$

The reconstructed victim column is

$$\widehat{W}_{\text{vic}, i} = \frac{W_{\text{obf}, \widehat{\pi}(i)} - \widehat{B}\widehat{c}_i}{\widehat{p}_i}. \quad (34)$$

Thus, the matching and reconstruction logic is unchanged, but successful recovery additionally requires an accurate rank and mask-subspace estimate.

F. Computational Complexity

For a d -by- n weight matrix, the attack has three dominant computational costs: dense SVD, pairwise projected cosine-similarity construction, and one-to-one assignment. Computing the dense SVD of the centered exposed matrix takes

$$O(dn \min\{d, n\}) \quad (35)$$

time. Constructing the projected cosine-similarity matrix requires $O(n^2d)$ time, and solving the resulting one-to-one assignment with the Hungarian algorithm requires $O(n^3)$ time.

Projection and per-column coefficient fitting require $O(dn\widehat{r})$ and $O(nd\widehat{r}^2)$ time, respectively. For the small mask ranks targeted by the scheme, these terms are lower order than the dense similarity construction and assignment. The overall runtime is therefore

$$O(dn \min\{d, n\} + n^2d + n^3). \quad (36)$$

The overall space complexity is $O(dn + n^2)$ for the input, projected, and similarity matrices.

V. RECOVERY GUARANTEES AND FAILURE CONDITIONS

A key step in the attack presented in Section IV is to estimate the shared mask direction from the exposed weights. This estimate underpins the subsequent mask-removal, column-matching, and victim-weight reconstruction stages. In this section, we analyze when and how accurately the shared direction can be estimated. We first establish sufficient conditions under which the rank-one mask signal dominates the residual spectrum, and then translate this spectral separation into bounds on the estimated direction and the resulting projection operator. Finally, we extend the analysis to rank- r masking and discuss the corresponding failure conditions. Throughout this section, quantities unavailable to the attacker, including $\widetilde{X}$, $\mathbf{v}$, and $\widetilde{\mathbf{q}}$, are introduced only to state and analyze the guarantees; the attack itself continues to use only W_{pre} and W_{obf} .

A. Dominance of the Rank-One Mask Signal

To characterize when the shared mask direction can be accurately estimated, we compare the rank-one mask signal with the remaining terms in the Gram matrix of the centered exposed weights. Recall from Eq. (13) that

$$\widetilde{W}_{\text{obf}} = (\widetilde{X} + \mathbf{v}\widetilde{\mathbf{q}}^\top)\Pi. \quad (37)$$

Because $\Pi\Pi^\top = I$,

$$G := \widetilde{W}_{\text{obf}}\widetilde{W}_{\text{obf}}^\top = \lambda_{\text{mask}}\mathbf{u}\mathbf{u}^\top + R, \quad (38)$$

where

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|_2}, \quad \lambda_{\text{mask}} = \|\widetilde{\mathbf{q}}\|_2^2 \|\mathbf{v}\|_2^2, \quad (39)$$

and

$$R = \widetilde{X}\widetilde{X}^\top + \widetilde{X}\widetilde{\mathbf{q}}\mathbf{v}^\top + \mathbf{v}\widetilde{\mathbf{q}}^\top\widetilde{X}^\top. \quad (40)$$

Let $\delta = \|R\|_2$.

Proposition (Rank-One Spectral Dominance). Suppose the centered mask coefficients have constant-order empirical variance, $\|\mathbf{v}\|_2^2 = \Omega(n)$ (that is, $\mathbf{v}$ aggregates a non-vanishing fraction of victim columns without catastrophic cancellation), and the scaled victim columns have bounded average squared norm. Then

$$\lambda_{\text{mask}} = \Omega(n^2), \quad \frac{\delta}{\lambda_{\text{mask}}} = O(n^{-1/2}).$$

In particular, a superlinear lower bound on $\|\mathbf{v}\|_2^2$ only strengthens the same ratio.

Proof: The first condition gives

$$\|\widetilde{\mathbf{q}}\|_2^2 = \sum_{i=1}^n (q_i - \bar{q})^2 = \Theta(n). \quad (41)$$

Combined with the hypothesis $\|\mathbf{v}\|_2^2 = \Omega(n)$ and Eq. (39),

$$\lambda_{\text{mask}} = \|\widetilde{\mathbf{q}}\|_2^2 \|\mathbf{v}\|_2^2 = \Omega(n^2). \quad (42)$$

For the scaled victim component,

$$\|X\|_F^2 = \sum_{i=1}^n \|p_i \mathbf{w}_i\|_2^2 = O(n). \quad (43)$$

Since column centering is an orthogonal projection,

$$\|\widetilde{X}\|_2 \leq \|\widetilde{X}\|_F \leq \|X\|_F = O(\sqrt{n}). \quad (44)$$

By submultiplicativity,

$$\delta \leq \|\widetilde{X}\|_2^2 + 2\|\widetilde{X}\|_2 \|\widetilde{\mathbf{q}}\|_2 \|\mathbf{v}\|_2. \quad (45)$$

Dividing by $\lambda_{\text{mask}} = \|\widetilde{\mathbf{q}}\|_2^2 \|\mathbf{v}\|_2^2$ yields

$$\frac{\delta}{\lambda_{\text{mask}}} \leq \frac{\|\widetilde{X}\|_2^2}{\|\widetilde{\mathbf{q}}\|_2^2 \|\mathbf{v}\|_2^2} + \frac{2\|\widetilde{X}\|_2}{\|\widetilde{\mathbf{q}}\|_2 \|\mathbf{v}\|_2}. \quad (46)$$

The first term is $O(n)/(\Theta(n) \cdot \Omega(n)) = O(n^{-1})$. The second term is $O(\sqrt{n})/(\Theta(\sqrt{n}) \cdot \Omega(\sqrt{n})) = O(n^{-1/2})$. Hence

$$\frac{\delta}{\lambda_{\text{mask}}} = O(n^{-1/2}). \quad (47)$$

The $O(n^{-1/2})$ rate predicts stronger spectral separation for wider protected matrices. Because the bound is asymptotic and conditional, we measure $\delta/\lambda_{\text{mask}}$ directly for each protected layer in Section VI.

B. Recovery of the Mask Direction

The preceding proposition bounds the residual relative to the rank-one mask signal. We now translate this spectral separation into an angular error bound for the estimated mask direction. Let $\hat{\mathbf{u}}$ be the unit leading left singular vector of $\widetilde{W}_{\text{obf}}$, equivalently a leading eigenvector of G . Define its angle from the true mask direction, up to sign, as

$$\theta = \arccos(|\hat{\mathbf{u}}^\top \mathbf{u}|) \in [0, \pi/2]. \quad (48)$$

Proposition (Mask-Direction Recovery). Under the decomposition in Eq. (38), the angular error of the leading left singular direction satisfies

$$\sin^2 \theta \leq \frac{2\delta}{\lambda_{\text{mask}}}.$$

Proof: Because $G = \widetilde{W}_{\text{obf}} \widetilde{W}_{\text{obf}}^\top$ is a symmetric positive-semidefinite Gram matrix, the Rayleigh–Ritz characterization implies that its unit leading eigenvector $\hat{\mathbf{u}}$ maximizes the Rayleigh quotient. Because $\mathbf{u}$ is also a unit vector,

$$\hat{\mathbf{u}}^\top G \hat{\mathbf{u}} = \lambda_{\max}(G) \geq \mathbf{u}^\top G \mathbf{u}. \quad (49)$$

Substituting Eq. (38) and using $\|\mathbf{u}\|_2 = 1$ gives

$$\begin{aligned} \lambda_{\text{mask}}(\hat{\mathbf{u}}^\top \mathbf{u})^2 + \hat{\mathbf{u}}^\top R \hat{\mathbf{u}} \\ \geq \lambda_{\text{mask}} + \mathbf{u}^\top R \mathbf{u}. \end{aligned} \quad (50)$$

Rearranging and applying $|\mathbf{x}^\top R \mathbf{x}| \leq \|R\|_2 = \delta$ for every unit vector $\mathbf{x}$ gives

$$\begin{aligned} \lambda_{\text{mask}}(1 - (\hat{\mathbf{u}}^\top \mathbf{u})^2) &\leq \hat{\mathbf{u}}^\top R \hat{\mathbf{u}} - \mathbf{u}^\top R \mathbf{u} \\ &\leq 2\delta, \end{aligned} \quad (51)$$

Since $\sin^2 \theta = 1 - (\hat{\mathbf{u}}^\top \mathbf{u})^2$,

$$\sin^2 \theta \leq \frac{2\delta}{\lambda_{\text{mask}}}. \quad (52)$$

Combining this bound with Eq. (47) gives $\sin \theta = O(n^{-1/4})$. Thus, under the stated scaling conditions, wider protected matrices admit a tighter direction-recovery bound.

C. Recovery of the Mask-Removal Projector

The attack uses the estimated mask direction through its orthogonal-complement projector. We therefore translate the angular error in $\hat{\mathbf{u}}$ into an operator-norm error for the resulting mask-removal projection. Following Section IV-C, define

$$P_{\mathbf{v}^\perp} = I - \mathbf{u}\mathbf{u}^\top, \quad P_{\hat{\mathbf{u}}^\perp} = I - \hat{\mathbf{u}}\hat{\mathbf{u}}^\top. \quad (53)$$

Proposition (Mask-Removal Projector Recovery). The true and estimated mask-removal projectors satisfy

$$\begin{aligned} \|P_{\hat{\mathbf{u}}^\perp} - P_{\mathbf{v}^\perp}\|_2 &= \sin \theta \\ &\leq \sqrt{\frac{2\delta}{\lambda_{\text{mask}}}}. \end{aligned}$$

Proof: Because $P_{\hat{\mathbf{u}}^\perp}$ is invariant to the sign of $\hat{\mathbf{u}}$, choose the sign so that $c = \hat{\mathbf{u}}^\top \mathbf{u} = \cos \theta \geq 0$, and let $s = \sin \theta = \sqrt{1 - c^2}$. If $s = 0$, then $\hat{\mathbf{u}} = \mathbf{u}$ under this sign choice, so the two projectors are identical. Otherwise, define

$$\mathbf{z} = \frac{\hat{\mathbf{u}} - c\mathbf{u}}{s}. \quad (54)$$

Since

$$\mathbf{u}^\top (\hat{\mathbf{u}} - c\mathbf{u}) = c - c = 0, \quad \|\hat{\mathbf{u}} - c\mathbf{u}\|_2^2 = 1 - c^2 = s^2, \quad (55)$$

$\mathbf{z}$ is a unit vector orthogonal to $\mathbf{u}$, and $\hat{\mathbf{u}} = c\mathbf{u} + s\mathbf{z}$. In the orthonormal basis $\{\mathbf{u}, \mathbf{z}\}$,

$$\begin{aligned} P_{\hat{\mathbf{u}}^\perp} - P_{\mathbf{v}^\perp} &= \mathbf{u}\mathbf{u}^\top - \hat{\mathbf{u}}\hat{\mathbf{u}}^\top \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} c^2 & cs \\ cs & s^2 \end{bmatrix} \\ &= \begin{bmatrix} s^2 & -cs \\ -cs & -s^2 \end{bmatrix}. \end{aligned} \quad (56)$$

This matrix has eigenvalues s and $-s$ and is zero outside $\text{span}\{\mathbf{u}, \mathbf{z}\}$. Hence,

$$\begin{aligned} \|P_{\hat{\mathbf{u}}^\perp} - P_{\mathbf{v}^\perp}\|_2 &= s = \sin \theta \\ &\leq \sqrt{\frac{2\delta}{\lambda_{\text{mask}}}}. \end{aligned} \quad (57)$$

where the inequality follows from Eq. (52). ■

Consequently, for any weight column $\mathbf{x}$,

$$\|P_{\hat{\mathbf{u}}^\perp} \mathbf{x} - P_{\mathbf{v}^\perp} \mathbf{x}\|_2 \leq \sqrt{\frac{2\delta}{\lambda_{\text{mask}}}} \|\mathbf{x}\|_2. \quad (58)$$

This bound controls the additional column-wise error introduced by the estimated projector. Exact permutation recovery additionally requires nonzero projected column norms and a positive matching margin, as formalized in Appendix A.

D. Rank- r Extension and Failure Conditions

For a centered rank- r mask $B\tilde{C}^\top$, where the columns of B are orthonormal, the mask contribution to the Gram matrix is

$$B(\tilde{C}^\top \tilde{C})B^\top. \quad (59)$$

Its nonzero eigenspace is $\text{span}(B)$. The proof follows the same steps as the rank-one case: replace the mask direction by the top- r mask subspace and compare its weakest nonzero eigenvalue with the residual spectrum. When this eigengap dominates the residual, the estimated and true subspace projectors are close.

Across both the rank-one and rank- r settings, recovery becomes unreliable when the mask signal is not sufficiently separated from the residual or when the projected columns lose their separation. Rank- r masking introduces additional

failure modes: the rank may be misestimated, the weakest mask direction may be too weak, or r may be so large that projection removes most of the matching information.

VI. EVALUATION

We evaluate *ArrowRevelio* as an end-to-end attack against *ArrowCloak*. This section focuses on the three research questions that establish the attack’s effectiveness, mechanism, and implications for *ArrowCloak*’s security premise:

- **RQ1: End-to-End Recovery.** Can *ArrowRevelio* recover the hidden permutation and reproduce victim functionality using only the public pre-trained weights and the obfuscated weights exposed outside the TEE?
- **RQ2: Recovery Mechanism.** Does the shared mask expose a recoverable low-rank subspace, and is the estimated mask-removal projector accurate enough to enable the permutation recovery underlying the end-to-end attack in RQ1?
- **RQ3: Error-Distribution Security Premise.** Do real fine-tuning residuals satisfy the normality and standard-deviation conditions used to support *ArrowCloak*’s claimed LWE mapping, and does *ArrowRevelio* remain effective when those conditions hold?

We also report the attacker-side runtime in this section. Additionally, we answer **RQ4: Higher-Rank Masking as Mitigation** in Section VII-B, where we examine how increasing the mask rank affects structural permutation recovery and functional victim recovery. We report **RQ5: Robustness to Attacker Uncertainty** last as a supplementary evaluation in Appendix C, since it concerns how an attacker can infer the mask rank and select a compatible public reference rather than the core recovery result.

A. Experimental Setup

Models and workloads. We evaluate six model–task pairs: ViT-B/16 [29] on CIFAR-100 [30], BERT-base [31] on SST-2 [32], GPT-2 [33] on SST-2, ViT on Kvasir-SEG [34], and SD 2.1 [35] LoRA on the Jewelry and Emoji datasets. We fine-tune ViT-B/16 on CIFAR-100 for 10 epochs (3,910 optimization steps) with an initial learning rate of 10^{-3} . BERT-base and GPT-2 are fine-tuned on SST-2 for three epochs (6,315 steps) with initial learning rates of 2×10^{-5} and 10^{-5} , respectively. The Kvasir-SEG model is trained for 12 epochs (1,200 steps), using learning rates of 3×10^{-4} for the segmentation decoder and 3×10^{-5} for the ViT backbone. For SD 2.1, we use the released Jewelry LoRA trained for 500 steps with a learning rate of 10^{-4} and the released Emoji LoRA checkpoint at 1,000 steps, whose initial learning rate is 10^{-4} . For each pair, W_{pre} denotes the public checkpoint used to initialize fine-tuning, and W_{vic} denotes the resulting victim checkpoint. We apply *ArrowCloak* independently to each protected linear layer and record the corresponding W_{obf} exposed to the GPU.

Defense and attack settings. Unless otherwise stated, we instantiate *ArrowCloak*’s original rank-one construction using

TABLE II
STRUCTURAL RECOVERY ACROSS SIX RANK-ONE PROTECTED-MODEL CONFIGURATIONS.

Configuration	Perm. acc. (%) $\uparrow$	Mean true-match rank $\downarrow$
ViT-B/16/CIFAR-100	99.95	1.27
BERT-base/SST-2	100.00	1.00
GPT-2/SST-2	100.00	1.00
ViT/Kvasir-SEG	99.92	1.31
SD 2.1/Jewelry	100.00	1.0034
SD 2.1/Emoji	100.00	1.0000

the exact coefficient distributions in its released implementation. The transformation secrets are resampled independently for each protected layer.

For each protected matrix, *ArrowRevelio* receives only W_{pre} and W_{obf} . The victim weights and *ArrowCloak*’s transformation secrets are used only for evaluating recovery and are never provided to the attack. We keep all attack hyperparameters fixed across comparable experiments. Rank- r experiments vary only the defense-side mask rank.

We run all experiments on an AMD machine, with EPYC 9115 16-core CPU and an NVIDIA RTX 6000 GPU.

B. RQ1: End-to-End Recovery

We first evaluate whether the structural recovery achieved by *ArrowRevelio* translates into an end-to-end compromise of real protected models. We apply the complete attack to six protected-model configurations spanning classification, segmentation, and diffusion.

We use permutation accuracy as the primary structural endpoint. It measures the fraction of protected weight vectors assigned to their ground-truth positions by the final one-to-one assignment. We additionally report mean true-match rank. For each protected reference vector, we rank all exposed candidates by their matching scores and record the position of the ground-truth match. A mean rank close to one indicates that the true match remains among the highest-scoring candidates even when exact recovery fails.

Table II shows that *ArrowRevelio* recovers the hidden permutation almost perfectly across all six configurations, with accuracy ranging from 99.92% to 100%. Mean true-match rank ranges from 1.00 to 1.31, showing that the ground-truth counterparts remain near the top of the candidate rankings even in the few cases where the final assignment is incorrect.

Having established structural recovery, we next examine whether the reconstructed weights reproduce the victim models’ functionality. For the three classification models, we report victim accuracy, recovered-model accuracy, and prediction agreement with the victim. For Kvasir-SEG, we report the victim and recovered models’ Dice and IoU against the ground-truth segmentation masks, together with agreement between their outputs. All three metrics are reported as percentages, with higher values indicating better performance or agreement. The two diffusion configurations do not provide a directly comparable task-performance benchmark. We therefore evaluate only victim–recovered output similarity, using PSNR and

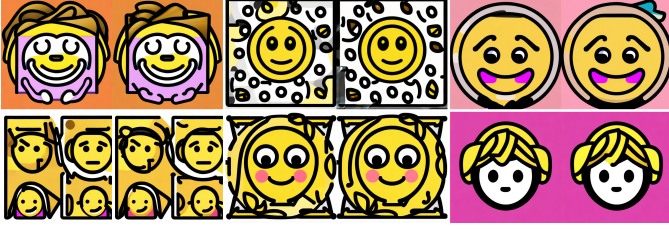


Fig. 3. Qualitative victim–recovered comparison on SD 2.1/Emoji under matched prompts and random seeds. Within each pair, the victim output is shown on the left and the recovered-model output on the right.

TABLE III

FUNCTIONAL RECOVERY FOR CLASSIFICATION AND SEGMENTATION. WE REPORT TASK PERFORMANCE FOR THE VICTIM AND RECOVERED MODELS TOGETHER WITH THEIR OUTPUT AGREEMENT.

Configuration (metric)	Victim	Rec.	Agree. (%)
ViT/CIFAR-100 (Acc.)	92.37	90.78	94.39
BERT-SST-2 (Acc.)	91.63	91.86	99.54
GPT-2/SST-2 (Acc.)	90.83	91.06	97.94
Kvasir-SEG (Dice/IoU)	89.61/83.00	87.58/80.12	98.35

SSIM on generations produced with matched prompts and random seeds to measure pixel-level fidelity and structural similarity, respectively.

Table III shows that classification accuracy differs from the victim by at most 1.59 percentage points, while prediction agreement ranges from 94.39% to 99.54%. On Kvasir-SEG, the recovered model is within 2.03 percentage points in Dice and 2.88 percentage points in IoU of the victim, with 98.35% output agreement. For diffusion, matched victim–recovered generations yield a PSNR of 18.5224 and an SSIM of 0.6125 on Jewelry, and a PSNR of 14.4303 and an SSIM of 0.8530 on Emoji.

Figure 3 provides a qualitative view of the Emoji results. Across the six examples, the recovered model preserves the main emoji subject. Differences remain in fine-grained details such as accessories, background patterns, and exact contour placement. Such discrepancies can arise from small residual recovery errors that propagate and may compound over successive denoising steps. Nevertheless, the strong structural similarity demonstrates the effectiveness of our recovery in reproducing the victim model’s main visual characteristics.

Results (RQ 1): *ArrowRevelio* achieves near-perfect permutation recovery across all six configurations, completes the structural-to-functional recovery chain for classification and segmentation, and quantifies victim–recovered output similarity for diffusion.

C. RQ2: Recovery Mechanism

We next examine why the end-to-end recovery in RQ1 succeeds. Section V predicts that shared-mask reuse produces a spectrally dominant rank-one component, causing the residual-to-mask ratio $\delta/\lambda_{\text{mask}}$ to decrease as $O(n^{-1/2})$ under the stated non-cancellation and bounded-energy conditions.

TABLE IV

REALIZED ACCURACY OF SHARED-MASK SUBSPACE RECOVERY. RESULTS ARE REPORTED AS MEDIAN (MAXIMUM) ACROSS PROTECTED LAYERS.

Model/task	Spectral ratio $\downarrow$ $\delta/\lambda_{\text{mask}} (\times 10^{-4})$	Direction error $\downarrow$ $\theta(\hat{u}, u) (^{\circ})$	Projector error $\downarrow$ $\ \hat{P} - P\ _2 (\times 10^{-4})$
ViT-B/16/CIFAR-100	2.88 (5.41)	0.0157 (0.0283)	2.75 (4.95)
BERT-base/SST-2	2.92 (5.17)	0.0160 (0.0287)	2.79 (5.01)
GPT-2/SST-2	2.84 (5.90)	0.0154 (0.0319)	2.69 (5.57)

We test whether this spectral separation materializes in real protected layers, whether *ArrowRevelio* accurately recovers the resulting mask subspace, and whether the estimated mask-removal projector is sufficiently accurate to support the Hungarian matching stage used in RQ1.

We evaluate this recovery chain independently on every protected matrix of ViT-B/16, BERT-base, and GPT-2. We measure three successive quantities in the recovery chain. First, the residual-to-mask spectral ratio $\delta/\lambda_{\text{mask}}$ measures whether the shared-mask signal dominates the remaining matrix structure. Second, the principal angle $\theta(\hat{u}, u)$ measures how accurately the leading singular direction recovers the true mask span. Third, defining $P = P_{v^{\perp}}$ and $\hat{P} = P_{\hat{u}^{\perp}}$, $\|\hat{P} - P\|_2$ measures the error in the mask-removal operation used before permutation recovery.

Table IV shows that the shared-mask component is sharply separated from the residual structure. The residual-to-mask ratio is below 6×10^{-4} even in the worst layer. Consistent with the analysis, the recovered direction deviates from the true mask direction by at most 0.032° , and the estimated and oracle mask-removal projectors differ by at most 5.6×10^{-4} in operator norm.

Appendix B separately verifies that the evaluated layers exhibit neither substantial cancellation in the shared mask vector nor growth in average victim-column energy, consistent with the finite-width premises of our analysis.

Results (RQ 2): Shared-mask reuse creates a stable, spectrally identifiable subspace. *ArrowRevelio* therefore closely approximates the oracle mask-removal projector, producing projected similarities that support near-perfect global permutation recovery.

D. RQ3: Error-Distribution Security Premise

To justify treating the fine-tuning residual $E = W_{\text{pre}} - W_{\text{vic}}$ as the LWE error term, *ArrowCloak* empirically checks whether E is approximately Gaussian and satisfies a stated standard-deviation threshold. We evaluate these error-distribution conditions in two complementary settings. First, for real victim residuals, we test raw and quantized normality and apply the stated standard-deviation threshold after quantization. Second, we construct a controlled Gaussian- E victim to test whether satisfying these conditions prevents permutation recovery.

Real victim residuals. *ArrowCloak* assumes that E follows a Gaussian distribution and uses its quantized form in the

TABLE V
ERROR-DISTRIBUTION CHECKS ON REAL FINE-TUNING RESIDUALS.

Model/task	Layers	Raw normal	Quant. std threshold	Quant. normal
ViT-B/16/CIFAR-100	48	1/48	48/48	1/48
BERT-base/SST-2	73	73/73	73/73	73/73
GPT-2/SST-2	48	39/48	48/48	39/48
ViT/Kvasir-SEG	48	6/48	48/48	5/48
SD 2.1/Jewelry	128	0/128	128/128	0/128
SD 2.1/Emoji	128	0/128	128/128	0/128

claimed LWE mapping. We therefore examine the residual both before and after quantization.

Table V reports how many protected layers pass each error-distribution check. Raw normal applies *ArrowCloak*’s normality check to the real-valued residual. Quant. normal tests the Gaussian-shape requirement after quantization, while Quant. std threshold tests whether the quantized residual satisfies the stated condition $\sigma > 2\sqrt{n}$.

All protected layers satisfy the quantized standard-deviation threshold, but the normality results vary substantially across models. Raw normality holds in many BERT and GPT-2 layers but few layers in other models. BERT satisfies both quantized checks in all 73 layers (72 Transformer linear matrices and the SST-2 classification head), and GPT-2 does so in 39 of 48 layers. In contrast, the remaining real victims satisfy quantized normality in few or no protected layers. Thus, the error-distribution conditions are not consistently satisfied across real victims.

Controlled Gaussian- E victim. We construct a controlled victim from ViT-B/16 on CIFAR-100. For each of its 48 layers, we sample a Gaussian residual E and set $W_{\text{vic}} = W_{\text{pre}} - E$. This experiment tests whether the Gaussian and variance properties of E alone prevent permutation recovery.

All 48 synthetic residuals satisfy raw normality and the stated standard-deviation threshold, while 42 of 48 satisfy both quantized checks. Nevertheless, *ArrowRevelio* achieves 99.914% permutation recovery, with a mean true-match rank of 1.2507. Thus, constructing the victim with a Gaussian residual does not remove the shared structure exploited by the attack. RQ1 provides a corresponding real-victim example in BERT: all 73 protected layers satisfy both quantized conditions, yet *ArrowRevelio* achieves 100% permutation recovery and 99.54% prediction agreement.

Results (RQ 3): The error-distribution conditions invoked by *ArrowCloak* are neither consistently satisfied by real victims nor sufficient to prevent recovery by *ArrowRevelio* when they are satisfied.

E. Attack Cost

We measure the end-to-end runtime of the complete attack on BERT-base with SST-2 while varying the mask rank. The attack takes 11.531 seconds at $r = 1$ and 17.170 seconds at $r = 32$, with intermediate settings falling within this range.

These results show that the attack remains computationally practical across the evaluated mask ranks.

VII. SECURITY IMPLICATIONS AND MITIGATION

The evaluation shows that *ArrowCloak*’s original rank-one masking remains vulnerable: *ArrowRevelio* achieves near-perfect permutation recovery and closely reproduces victim functionality. This observation reveals a shift from local to cross-vector leakage and motivates a natural mitigation: increasing the mask rank. We first discuss the broader security implications of this shift and then evaluate the resulting security–efficiency tradeoff.

A. Local and Cross-Vector Leakage

Our attack, *ArrowRevelio*, recovers the hidden permutation and reconstructs victim functionality even after *ArrowCloak* disrupts the per-vector directional similarity exploited by *ArrowMatch*. This result complements *Game of Arrows* rather than extending the same leakage mechanism: *ArrowMatch* exploits local geometry preserved within individual vectors by scaling and permutation, whereas *ArrowRevelio* exploits cross-vector structure created when *ArrowCloak* reuses a shared mask direction, across the released vectors.

Together, the two attacks show that concealing one local invariant does not secure a joint release if the protection mechanism introduces dependencies across released values. Confidential-offloading defenses should therefore be evaluated under the complete accelerator view, including public initialization checkpoints, per-vector invariants, and dependencies across jointly released weights. Designs that amortize trusted work by reusing low-dimensional secret state should be tested against adaptive attacks that estimate or remove that state.

B. The Security–Efficiency Tradeoff of Higher-Rank Masking

ArrowCloak independently scales and permutes the victim-weight vectors and adds a mask to each vector. Although these masks disrupt the pairwise directional similarity exploited by *ArrowMatch*, they are not independent across the released vectors. In the original construction, every mask lies in one shared direction; in the rank- r extension, the masks lie in a shared r -dimensional subspace. This reuse creates a coherent matrix-level signal that can be estimated and removed when the subspace is spectrally visible.

Increasing the mask rank is thus a natural mitigation. A larger r leaves fewer complementary dimensions for matching and places a larger fraction of each victim-weight vector inside the ambiguous mask subspace. We therefore test whether increasing r can successfully disrupt structural recovery, functional recovery, or both.

Mitigation effectiveness. We evaluate high mask rank on BERT-base with SST-2, whose masking space has dimension 768. We vary the mask rank from 64 to 512 and apply the automatic rank estimator and the complete attack at each rank. We report the estimated rank, permutation accuracy, and prediction agreement with the victim.

TABLE VI
STRUCTURAL AND FUNCTIONAL RECOVERY UNDER HIGHER-RANK
MASKING ON BERT-BASE WITH SST-2; PERMUTATION ACCURACY AND
AGREEMENT ARE IN PERCENT.

True r	Est. $\hat{r}$	Perm. acc.	Agreement
64	64	100.00	95.53
128	128	100.00	90.02
256	256	100.00	74.43
512	512	99.98	48.05

Table VI shows that increasing the mask rank primarily degrades functional reconstruction rather than rank estimation or permutation matching. The automatic estimator returns the true rank at every tested setting, while permutation accuracy remains 100% through $r = 256$ and 99.98% at $r = 512$. This persistence follows from the coherent, spectrally identifiable structure of the rank- r mask. SVD recovers its dominant directions, and projecting them out leaves sufficient directional information for column matching. Even at $r = 512$, the complementary subspace retains 256 dimensions.

Functional recovery responds differently. Agreement with the victim decreases from 95.53% at $r = 64$ to 48.05% at $r = 512$, compared with 99.54% for the original rank-one configuration. Within the estimated mask subspace, the victim component is confounded with the additive mask. As r increases, this ambiguous component occupies a larger fraction of each weight vector, and the coefficient fit cannot recover a growing portion of the victim weights. Higher rank therefore leaves the structural correspondence observable but prevents it from supporting accurate functional reconstruction.

TEE computation overhead. The original rank-one design requires the TEE to apply one mask direction during each protected linear operation. A rank- r construction applies r directions, so the rank-dependent part of the TEE correction requires r times the arithmetic of its rank-one counterpart.

We normalize this cost against a no-offloading baseline in which all 72 protected BERT-base matrix multiplications execute inside the TEE. Under the original rank-one design, the correction requires only 0.20% of this baseline. For each protected matrix, the rank- r correction scales with r times the sum of its input and output dimensions, whereas full TEE execution scales with their product. Aggregating these operation counts over the 72 protected matrices gives a correction-to-full-TEE ratio of $r/512$.

At $r = 512$, the setting that reduces victim agreement to 48.05%, the TEE correction therefore requires approximately as much arithmetic as executing all protected matrix multiplications inside the TEE. This comparison reflects theoretical operation counts rather than measured latency, which depends on the TEE hardware and implementation. High-rank masking can thus suppress functional leakage, but at $r = 512$ it does so at the cost of increasing trusted computation inside TEE.

VIII. RELATED WORK

TEE-assisted inference systems place the GPU on one of two sides of the trust boundary. Graviton, Telekine, and NVIDIA Confidential Computing on H100 GPUs place GPU execution inside the trusted boundary and provide hardware-backed isolation and attestation [25], [26], [3]. CPU-TEE partitioning systems instead treat the GPU as untrusted: expensive linear operations run on the accelerator, whereas secrets and selected operations remain inside the CPU TEE [4]. Our attack targets the information exposed by this latter design choice, not the security of the TEE itself.

Systems in this setting differ in what they protect and how they transform inputs or model weights before outsourcing computation. Slalom protects private inputs and verifies outsourced linear operations [4]. SOTER additionally protects model parameters through parameter morphing and checks the untrusted execution using oblivious fingerprints [7]. ShadowNet transforms the weights of linear layers before exposing them to an accelerator, restores the corresponding outputs inside the TEE, and retains nonlinear layers within the trusted boundary [8]. These designs share a central efficiency constraint: weight transformation and output recovery on the trusted CPU must remain substantially cheaper than the matrix multiplication delegated to the GPU.

The closest prior security analysis is *Game of Arrows* [17], which shows that lightweight scaling and permutation preserve the directions of protected weight vectors. Its *ArrowMatch* attack uses a public pre-trained model to recover the hidden correspondence from this directional similarity and then uses query-labeled data to adjust the recovered vector lengths. The same work introduces *ArrowCloak*, which adds a shared mask direction to disrupt direct direction matching.

A recent concurrent preprint, *MOSAIC*, questions *ArrowCloak*'s LWE-based security argument by observing that replacing the fine-tuning residual with fresh independent Gaussian noise produces distinguishable outputs. This critique targets the reduction rather than the concrete scheme and does not by itself provide a practical weight- or key-recovery attack [21]. Our attack targets *ArrowCloak* itself. *ArrowRevelio* estimates and removes the shared low-rank mask subspace before recovering permutation. It reconstructs a functional surrogate without query-labeled data or knowledge of the secret transformation parameters. The analysis also extends from the published rank-one construction to rank- r masks.

IX. CONCLUSION

In this paper, we reassessed the LWE-based security claim of *ArrowCloak* and showed that its LWE-shaped reformulation does not transfer standard LWE hardness to model recovery. Examining the concrete construction, we found that reusing a shared mask subspace leaves a low-rank signal unchanged by the hidden permutation. We exploit this leakage with *ArrowRevelio*, a polynomial-time attack that uses only the public pre-trained and exposed obfuscated weights to estimate and remove the mask subspace, recover the hidden permutation, and reconstruct the victim weights. We establish sufficient

spectral-separation conditions for accurate recovery of the mask-removal projector. Our evaluation validates this leakage: across six protected-model configurations, *ArrowRevelio* recovers 99.92–100% of the hidden permutation and produces surrogate models that closely reproduce victim functionality.

ETHICS CONSIDERATIONS

This work examines the confidentiality guarantees of *ArrowCloak* and identifies a vulnerability that could enable the unauthorized recovery of proprietary models. Our experiments were conducted locally using public checkpoints and datasets and the released *ArrowCloak* implementation; we did not target deployed third-party systems, access private user data, or exploit TEE vulnerabilities. We report these findings to support independent security evaluation and the development of stronger confidential-inference defenses.

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APPENDIX A

MATCHING STABILITY AND PERMUTATION RECOVERY

Section V bounds the error of the estimated mask-removal projector. We now show when this error is small enough to preserve the projected cosine similarities and the hidden one-to-one correspondence.

Let $\mathcal{W}$ contain all nonzero public and exposed columns used in matching, and define

$$\eta = \|\hat{P} - P\|_2, \quad \rho = \min_{w \in \mathcal{W}} \frac{\|Pw\|_2}{\|w\|_2}. \quad (60)$$

The quantity ρ measures the smallest fraction of column norm retained after ideal mask removal. A positive ρ excludes columns that lie almost entirely in the mask direction.

Lemma 1 (Normalized projection stability): Assume $\rho > 0$ and $\eta < \rho$. Then, for every $w \in \mathcal{W}$,

$$\left\| \frac{\hat{P}w}{\|\hat{P}w\|_2} - \frac{Pw}{\|Pw\|_2} \right\|_2 \leq \frac{2\eta}{\rho}. \quad (61)$$

Proof: Let $a = Pw$ and $\hat{a} = \hat{P}w$. By Eq. (60),

$$\|\hat{a} - a\|_2 \leq \eta\|w\|_2, \quad \|a\|_2 \geq \rho\|w\|_2. \quad (62)$$

Moreover,

$$\|\hat{a}\|_2 \geq (\rho - \eta)\|w\|_2 > 0. \quad (63)$$

Thus both normalized vectors are well defined. The reverse triangle inequality gives

$$\begin{aligned} \left\| \frac{\hat{a}}{\|\hat{a}\|_2} - \frac{a}{\|a\|_2} \right\|_2 &\leq \frac{|\|a\|_2 - \|\hat{a}\|_2|}{\|a\|_2} + \frac{\|\hat{a} - a\|_2}{\|a\|_2} \\ &\leq \frac{2\|\hat{a} - a\|_2}{\|a\|_2} \leq \frac{2\eta}{\rho}. \end{aligned} \quad (64)$$

Define the oracle cosine score

$$S_{ij}^* = \cos(Pw_{\text{pre},i}, Pw_{\text{obf},j}), \quad (65)$$

which uses the unavailable true projector. Let S_{ij} be the estimated score in Eq. (21). Applying Lemma 1 to both normalized arguments of the cosine and adding and subtracting their mixed inner product gives

$$|S_{ij} - S_{ij}^*| \leq \frac{4\eta}{\rho}. \quad (66)$$

For public column i , define

$$\epsilon_i = \max_j |S_{ij} - S_{ij}^*| \leq \frac{4\eta}{\rho}, \quad (67)$$

and the oracle matching margin

$$\mu_i = S_{i,\pi(i)}^* - \max_{j \neq \pi(i)} S_{ij}^*. \quad (68)$$

Theorem 1 (Permutation recovery): If $\mu_i > 2\epsilon_i$ for every i , then the linear assignment in Eq. (22) uniquely recovers the hidden permutation π .

Proof: For every $j \neq \pi(i)$,

$$\begin{aligned} S_{i,\pi(i)} - S_{ij} &\geq S_{i,\pi(i)}^* - S_{ij}^* - 2\epsilon_i \\ &\geq \mu_i - 2\epsilon_i > 0. \end{aligned} \quad (69)$$

Thus, the correct exposed column is the unique highest-scoring match in every row of S . Because π is bijective, it is the unique permutation that simultaneously selects all row-wise maxima. The Hungarian algorithm therefore returns π . ■

Combining Eq. (67) with Eq. (57) gives the explicit sufficient condition

$$\mu_i > \frac{8}{\rho} \sqrt{\frac{2\delta}{\lambda_{\text{mask}}}} \quad \text{for every } i. \quad (70)$$

This condition separates the two requirements for exact matching: the estimated mask projector must be accurate, and the ideal projected columns must retain enough norm and pairwise separation.

APPENDIX B

FINITE-WIDTH CHECKS FOR SPECTRAL DOMINANCE

Section V predicts spectral dominance when the shared mask vector combines victim columns without substantial cancellation and the scaled victim columns have bounded average squared norm. These are sufficient conditions used to explain the realized recovery measurements in Table IV, rather than attack-success metrics by themselves. We examine whether they are consistent with the protected matrices at their deployed widths.

TABLE VII

FINITE-WIDTH CHECKS FOR THE SPECTRAL-DOMINANCE PREMISES. r_v AND c_v ARE THE MEDIAN (MINIMUM) OF PER-LAYER MEDIANS OVER 100 DEFENSE DRAWS; $\|X\|_F^2/n$ IS THE MEDIAN (MAXIMUM) ACROSS LAYERS.

Model/task	$r_v \uparrow$ med (min)	$c_v \uparrow$ med (min)	$\ X\ _F^2/n$ med (max)
ViT-B/16/CIFAR-100	2.49 (0.56)	0.99 (0.33)	0.98 (6.29)
BERT-base/SST-2	8.86 (1.36)	0.99 (0.36)	3.19 (12.22)
GPT-2/SST-2	92.30 (13.15)	0.96 (0.35)	32.58 (301.87)

For each protected matrix, we draw *ArrowCloak* coefficients $k_i \sim \text{Unif}\{0, 1, 2, 3, 4\}$, form $\mathbf{v} = W_{\text{vic}}\mathbf{k}$, and record

$$r_v = \frac{\|\mathbf{v}\|_2^2}{n}, \quad c_v = \frac{\|\mathbf{v}\|_2^2}{\sum_{i=1}^n k_i^2 \|\mathbf{w}_i\|_2^2}. \quad (71)$$

The normalized energy r_v checks whether $\|\mathbf{v}\|_2^2$ remains on the order of n at the evaluated widths. The cancellation ratio c_v compares the realized energy of the sum with the sum of the individual contribution energies; a value near zero would indicate that the victim columns largely cancel one another. We also measure $\|X\|_F^2/n$ for $X = W_{\text{vic}}D_1$, which is the average squared norm of the scaled victim columns.

Table VII shows that the shared direction retains non-negligible energy at the evaluated widths. Across all protected matrices, the minimum observed r_v is 0.56 and the minimum c_v is 0.33, providing no evidence of severe cancellation. The centered mask coefficients have empirical variance approximately 10, consistent with their constant-order variance in the analysis. Although GPT-2 has a larger absolute column scale, the within-backbone Spearman correlations between $\|X\|_F^2/n$ and n are -0.27 , -0.13 , and -0.25 for ViT, BERT, and GPT-2, respectively; thus, we observe no increase with width in these finite models. These measurements are consistent with the non-degeneracy premises at the deployed widths, but do not by themselves prove the asymptotic $\Omega(n)$ and $O(n)$ conditions.

APPENDIX C

SUPPLEMENTARY EVALUATION

A. RQ5: Robustness to Attacker Uncertainty

The rank- r extension automatically estimates the mask rank from the exposed spectrum, while correspondence matching requires a compatible public reference. RQ5 evaluates rank inference and reference selection using only the exposed weights and public information.

Unknown mask rank. We evaluate automatic rank inference on BERT-base with SST-2, whose masking space has dimension 768. For this evaluation, we instantiate the acceptance thresholds as $\tau_{\text{gap}} = 10$ and $\tau_{\text{sep}} = 1.5$. We vary the true mask rank over $r \in \{2, 4, \dots, 512\}$ and apply the automatic estimator described in Section IV-E independently to each protected layer.

Panel A of Table VIII shows that the estimator recovers the true rank in all 72 protected layers for every tested r . Even at $r = 512$, where the mask occupies two thirds of the

TABLE VIII

AUTOMATIC RANK INFERENCE AND END-TO-END RECOVERY ON BERT-BASE WITH SST-2; ACCURACIES ARE IN PERCENT.

Panel A: Rank-inference confidence			
True r	$\hat{r} = r$ layers	Minimum $g_{(1)}$	Minimum $g_{(1)}/g_{(2)}$
2	72/72	346.85	28.53
4	72/72	379.03	25.62
8	72/72	345.77	17.87
16	72/72	341.39	15.03
32	72/72	314.70	12.35
64	72/72	306.22	11.40
128	72/72	301.06	9.24
256	72/72	232.27	7.51
512	72/72	95.27	3.73

Panel B: Recovery with the inferred rank			
True r	$\hat{r}$	Perm. acc.	Recover acc.
2	2	100.00	91.51
4	4	100.00	91.74
8	8	100.00	92.20
16	16	100.00	91.28
32	32	100.00	91.97
64	64	100.00	90.37

768-dimensional space, the weakest layer has $g_{(1)} = 95.27$ and $g_{(1)}/g_{(2)} = 3.73$, well above the respective acceptance thresholds. Panel B evaluates the complete attack using these inferred ranks. It achieves 100% permutation accuracy for every end-to-end setting from $r = 2$ to $r = 64$, while recovered accuracy remains between 90.37% and 92.20%. Thus, the attack does not require oracle knowledge of the mask rank in the evaluated settings. RQ4 separately examines the defensive effect of increasing r on functional recovery.

Unknown public reference. The exposed tensor dimensions and GPU-side operations reveal the protected layer shapes, allowing the attacker to narrow the search to architecture-compatible public checkpoints. We evaluate reference selection using GPT-2 as a case study. For each candidate, we automatically estimate and remove the masking subspace, and then compare every projected exposed column with all columns in the candidate reference. Let $s_{l,i}^{(1)}$ and $s_{l,i}^{(2)}$ denote the largest and second-largest cosine similarities for column i in layer l . We define the reference-selection score as

$$S = \frac{\sum_l \sum_{i=1}^{n_l} (s_{l,i}^{(1)} - s_{l,i}^{(2)})}{\sum_l n_l}, \quad (72)$$

where n_l is the number of protected columns in layer l . A larger score indicates clearer and more consistent correspondences across the protected layers.

The exact GPT-2 initialization receives a selection score of 0.80472 and enables perfect permutation recovery. In contrast, the independently pretrained checkpoint receives a score of 0.00921, yields only 0.09% permutation accuracy, and reduces recovered accuracy to 51.83%. The selection score therefore

TABLE IX
PUBLIC-REFERENCE SELECTION AMONG ARCHITECTURE-COMPATIBLE
GPT-2 CHECKPOINTS; ACCURACIES ARE IN PERCENT.

Candidate	Relation	Selection score	Perm. acc.	Recover acc.
Exact GPT-2	Victim initialization	0.80472	100.00	91.06
Mission	Independent			
NoShuffle	pre-training	0.00921	0.09	51.83

separates compatible and incompatible protected backbones in this candidate set.

Results (RQ 5): *ArrowRevelio* is robust to both forms of uncertainty when the exposed spectrum contains a clear rank break and the candidate pool contains the correct protected backbone. The mask rank need not be known in advance. However, an independently pretrained checkpoint with the same architecture is not a sufficient substitute when the correct protected backbone is unavailable.